IMPACT FACTOR- 4.520 ISSN NO- 2456-7078

Volume-1, Issue-VI, Sept- 2017

Research maGma

An International Multidisciplinary Journal

CONFERENCE

of

Thiruvalluvar University College of Arts & Science, Arakkonam.

on

"EMERGING TRENDS IN ENGLISH LANGUAGE & LITERATURE"

Research maGma
An International Multidisciplinary Journal

ISSN NO- 2456-7078 IMPACT FACTOR- 4.520 VOLUME-1, ISSUE-7, SEPT-2017

A CASE STUDY ON STUDENTS' UNDERSTANDING OF THE UNIFORM CONVERGENCE OF SEQUENCES OF CONTINUOUS FUNCTIONS IN INDIA

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ABSTRACT

It is a very important question that degree college students, study the concepts in mathematics with understanding or not. In this research, author has investigated students' understanding of the definitions of sequence of continuous functions and its uniform convergence. The author selected three male and five female students out of the senior class of a university constituent college and conducted questionnaire surveystwo times and examined students' understanding of sequence of continuous functions and its uniform convergence and also how they grasp right concept definitions for those through several progressive questions. In addition, author has presented some suggestions for effective teaching-learning for the sequences of continuous functions.

KEYWORDS:

Sequence of Continuous functions, Uniform convergence, Point-wise convergence.

INTRODUCTION

The college level Calculus and Set theory courses precede the Real analysis course in general in the learning of mathematics majors in the university. Students of the Mathematics department are introduced to symbolized, formal, and abstract major subjects of college level through learning a set theory. Students get a foundation for the major subject through training of logical thinking by means of symbols in the study of Set Theory. If students are lacking in logical thinking about the various theories in the subject of Set Theory, they would have difficulty in learning a subject which requires a rigorous and formal thinking in terms of an existential quantifier $\exists$, an universal quantifier $\forall$, and a symbol $\in$ in the upper class subject, Advanced Calculus in particular, with which this research is concerned. In this research, we investigated students' understanding of sequences of continuous functions through analyzing the concept images that students acquired.

Dieudonne (1969) intentionally tried not to use pictures asserting that mathematics should be founded on the axiomatic methods not depending on geometrical intuitions. Many of the contemporary mathematicians insisted the incompatibility of the Mathematical rigor and the employment of pictures based on the geometrical intuitions. However, considering the assertions (Brown, 1999) that pictures still brought forth crucial ideas for the major mathematical discoveries, we think that the use of pictures may not be excluded in the learning of plane curves. But, after recognizing the limitations of the geometrical intuition by the existence of a non-differentiable function which is continuous everywhere due to Weierstrass, the $\in - \delta$ proof due to Bolzano and Weierstrass has come to be regarded as a rigorous way of explaining continuity of functions in the mathematical com-munity. Nevertheless, when we investigate students' understanding of the mathematical concepts, it is important for a better teaching-learning of mathematical concepts explained in terms of $\in - \delta$ or $\in - N$ to observe the personal images which learners have on the $\in - \delta$ definition and $\in - \delta$ proof.

Students who participated in this research lacked in understanding the sequences of continuous functions since the concept of uniform convergence of sequences is contained in the later chapters of the textbook of Advanced Calculus used in their department and usually tended to be treated inadequately for lack of time. Therefore, it may be regarded as an appropriate research subject to investigate students' understanding of the sequences of continuous functions through their concept images. We also expected students had a chance to get right concept definition through participation in our surveys. Furthermore, we wanted to grasp students' difficulty in understanding the concept of uniform convergence at their level since we found that many of math students seem to share the same difficulty in the classes of Real Analysis.

THEORETICAL DEFINITIONS:

The definitions of point-wise convergence and the uniform convergence of sequencesof functions are briefly discussed here; Let $D \subseteq R$ and a function $f_n : D \to □$ be defined for each $n \in □$ where □ is the set of real numbers. Then the set $\{f_n\}$ of such functions is called a sequence of real functions. If $\{f_n(x)\}$ converges for each $x \in D$ when $n \to \infty$, we say that $\{f_n\}$ converges point-wise. In this case by the uniqueness of the limit, the limit function $f : D \to □$ is defined by $f(x) = \lim_{n \to \infty} f_n(x), \quad x \in D$ By the $\in - N$ definition of limit, if there exist a positive integer N such that $N = N(x, \in)$ for each $x \in D$ and for an arbitrarily $\in > 0$ such that $|f_n(x) - f(x)| < \in \ \forall n \geq N$, then we say that $\{f_n\}$ converges point-wise to f, and denoted by $f_n \to f$. The additional condition of uniform convergence is required to guarantee continuity of the limit function of a (point-wise) convergent sequence of continuous functions. If for a function $f : D \to □$ there exists a positive integer $N = N(\in)$ for an arbitrarily given $\in > 0$ such that $|f_n(x) - f(x)| < \in \ \forall n \geq N$ and for all $x \in D$, then we say that $\{f_n\}$ converges uniformly to f. If a sequence of continuous functions defined on a set converges uniformly, the limit function is continuous. In the case when the set *D* is a close dinterval, a given sequence of continuous functions converges uniformly if its limit function is continuous (T Apostol, 2002).

METHOD OF RESEARCH:

We selected 8 students as participants in this research whose grades belong to the top 30 percent of the senior class in the mathematics department in Thiruvalluvar University College of Arts and Science Arakkonam. We call the 3male students of the participating students as A, B, C, and the remaining 5 female students as D, E, F, G and H. For the purpose of investigating any differences in the results of surveys by gender, students were separated into the male and female groups which we called M and F respectively and the surveys were conducted separately. The surveys were conducted two times from July 12 th to Aug 4th in 2017 as shown below.

Table 1: Questionnaires Schedule

S.No	Date & Time	Contents of questionnaires	Duration
1	July 21st& 4th Period	Students' background & Learning attitude. Students' understanding of Definitions of sequences of continuous functions.	1 Hour
2	August 4th& 3rd Period	Students' understanding of convergence of sequences of continuous functions.	1 Hour

CONTENTS OF THE RESEARCH:

The students chosen for this study have taken the same course in the second semester of 2016 and learned the following concepts from "Mathematical Analysis" by Tom Apostol (2002) which is being the prescribed text book for the course. In chapter 4, limits and continuity: limit of a function, continuous functions, uniform continuity, monotone functions and discontinuities,' and in chapter 9, 'sequences and series of functions: point-wise convergence, uniform convergence, uniform convergence and continuity. Therefore, the background of students may be equal on some of the contents of Real Analysis 2, namely, on the limits and convergence of continuous functions and sequences of continuous functions with which we are concerned in this research. Since the researchers have experienced that many of the students would go through difficulty in this concept while giving lectures of Real Analysis 1.

FIRST QUESTIONNAIRE AND ANALYSIS:

The participating students were grouped in to A, B, C (Male Students) and D, E, F, G, H (Female Students)and the results of the survey are summarized in Table 2. All of the 8 participating students have taken almost all courses of the major subjects which are offered by department of mathematics including subjects of Real analysis 2, Differential Geometry, General Topology Complex analysis 1.

In table 2, 1 represents very good, 2 good, 3 average, 4 not good. The results show that students participating in the survey generally prefer General Topology to Real analysis 2, and got better grades in General Topology. While students' understanding of sequences of continuous functions is normal on average, their conceptual understanding of limit of sequences of continuous functions is closer to 'good acquaintance'. The geometric approach is given a little higher preference to the analytic approach, and students responded that when encountering a function, they try to associate a graph of it. In the following, students' responses to Questions 8 and 9 are described.

Question 8:Do you prefer geometric intuitions to an analytic approach? If so, what is the reason?

A - B: I prefer an analytic approach involving some analyses

B-H: I prefer a geometric approach since geometrical intuitions facilitate understanding

Question 9:Describe your present concept image of a sequence of functions.

Students vaguely associate properties and limits of sequences of functions as well as the definitions of it.

A: The rules of sequences of numbers and properties of functions will show up compositely.

B: A function is given to some sequence. Do the sequence have limit?
C: A function itself becomes the sum of a sequence which is connected with it in a certainway.
D, E, F: A sequence each of whose terms is a function.
G, H: A sequence which make use of functions

S No	Questions	A	B	C	D	E	F	G	H	%
1	I am familiar with the concept of continuous functions	3	2	2	1	2	1	1	1	4.2
2	I am familiar with the concept of sequences of continuous functions	3	3	4	3	3	2	3	3	3
3	I am familiar with the concept oflimit of sequences of functions	2	3	3	3	1	2	1	2	3.7
4	I like the subject of Real analysis.	3	2	2	1	1	2	1	1	3.7
5	What is your grade in Real analysis?	2	3	2	1	2	1	1	2	3.2
6	I like the subject of Topology.	1	2	2	2	3	2	2	2	2.8
7	When encountering a function,I try to associate its graph first ofall.	2	1	1	2	1	2	2	3	3
8	Do you prefer geometric intuitionsto an analytic approach?If so, what is the reason?	4	2	1	3	1	2	2	3	3.8
9	Describe your present understanding of a sequence of functions.	2	3	4	3	1	2	1	2	3.7

2) Students' understanding of concepts of sequences of continuous functions.

A questionnaire was conducted on the students and their response has been valued. After their response some suggestions also given.

First questionnaire—definitions of a sequence of continuous functions
Question 1. Write down the definition of a sequence of continuous functions.
Question 2. Write down the definition of a limit of a sequence of functions.
Question 3. Describe freely the visual image you have in your mind on the limit of a sequence of

CONTINUOUS FUNCTIONS

SECOND QUESTIONNAIRE AND ANALYSIS:

The second questionnaire in Table 7 was conducted to investigate students' understanding of the convergence of a sequence of continuous functions and the condition on continuity of the limit.

On the second questionnaire, male students A, B, and C tried to solve the problems without drawing graphs. On the other hand, except student E for Question 1, female students D, E, F, G and H drew the graphs in order to know the uniform convergence. Female students tried to solve the problems with visual images in mind. And those images are not given here.

Question 1. For $n \in \mathbb{N}$, let $f_n(x) = x^n$, $x \in [0, 1]$.

1. Is $\{f_n\}$ a sequence of continuous functions on [0, 1]?
2. Is the limit function continuous?
3. Does $\{f_n\}$ converge uniformly to on [0, 1]?
4. Does $\{f_n\}$ converge uniformly to on $[0, r]$ for any , $0 < r < 1$?
5. Is the limit function f continuous on [0, r] for any , $0 < r < 1$?

RESULTS:

In this research we have done case studies to examine and analyze how students ar-rived at the -proof via geometrical intuition. We developed the questionnaires stage by stage, and then carried out the surveys four times, before analyzing the research results to give us an accurate idea of how students conceptualized both continuous functions and sequences of continuous functions. These are areas that are typically difficult for students while studying Advanced Calculus and General Topology. As a result of this research, we have some suggestions on how to improve the teaching and learning methods regarding the convergence of continuous functions.

Through this research, we have identified some mathematical misconceptions about the point-wise convergence and the uniform convergence that students hold. When students' concept images are expressed through discussion or presentation in the regular class, learned knowledge can be updated. Similarly, students can nourish the democratic citizenship to accept various thoughts by experiencing community life via discussion. This is one of the goals of mathematics education (Kim, Park & Woo, 2007).

The fallacies that students committed in this questionnaire study are as follows: First, students showed a lack of understanding of the symbol '$\forall$'. In the second and third questionnaires, students thought that the difference between the point-wise convergence and the uniform convergence was to write 'each' and '$\forall$' in the same place of the definition, respectively (students B and C of Table 6, student D of Table 8). They did not pay attention to the location of the symbol '$\forall$' for the uniform convergence and they thought of the symbols '$\forall$, $\exists$' as 'there exists for the whole domain'. In some sense, this can be seen as a result of the differences in word order between Korean and English. Second, students thought that if the limit function was continuous, then the sequence of continuous functions must be uniformly convergent. Students tended to think that the sequences of functions converge uniformly regardless of the domain of sequences of functions.

SUGGESTION:

Based on the results of this questionnaire study, we suggest the following: First, various easy examples must be practiced for students to understand the symbol '$\forall$' completely. Second, since students know that if a sequence of continuous functions is uniformly convergent, then the limit function is continuous, they assume that the converse is also true and hence a sufficient explanation of the conditions needed for a converse proposition to be valid is needed. Third, the internalization by examples is needed for the fact that false propositions can be shown using a counterexample, but true propositions cannot be proved by an example. Fourth, students should be trained in order not to make logical fallacies when they use mathematical symbol and do verbal justification. Finally, emphasis should be placed on the importance of checking the domain of functions at any time.

As in the fourth questionnaire, students represented various concept images about the point-wise convergence and the uniform convergence. Appropriate visual examples will help students to attain concept definition of point-wise convergence and uniform convergence. However, as Hwang & Shim (2010) assert, such geometric intuition can cause intuitive obstacle. A concrete teaching strategy should be used to overcome this issue. As a teaching method for a proper balance between formal analysis and intuitive analysis of the concept, Lee & Park (2001) suggested harmony of intuition and logic, overcoming the phenomenon of functional fixation, development of useful intuitive model, development of meta-cognitive ability, and so on. Practice of strict formal inference is a necessary mental training exercise in order to attain the concept definition via the process of depersonalization beyond the concept image. Sufficient time for understanding thoughts, ideas, symbols, and concepts shared through small group discussion with peers can be a good teaching method in the process of depersonalization as well as personalization.

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